Math 141 **Test #1** **Fall 2017**

1. (16 pts)

   (a) Factor the quadratic polynomial:

   $$(3x - 1)(x + 4) = 0.$$

   Therefore, $3x - 1 = 0$ or $x + 4 = 0$, so $x = 1/3$ or $x = -4$.

   (b) Multiply through by $\sqrt{x^2 - 2}$ and get

   $$\begin{aligned}(x^2 - 2) - x &= 0,\\ x^2 - x - 2 &= 0,\\ (x - 2)(x + 1) &= 0.\end{aligned}$$

   Therefore, $x = 2$ or $x = -1$. But we must reject $x = -1$ because it does not satisfy the original equation.

2. (8 pts) Clearly, $A = 70°$. We also have

   $$\begin{aligned}\tan 20° &= \frac{b}{3},\\ b &= 3\tan 20°\\ &= 1.0919\end{aligned}$$

   and

   $$\begin{aligned}\cos 20° &= \frac{3}{c},\\ c &= \frac{3}{\cos 20°}\\ &= 3.1925.\end{aligned}$$

   Or, once you find that $b = 1.0919$, you could use the Pythagorean Theorem to find c.

3. (10 pts) It is very convenient to enter the formula as a function in the calculator. Then substitute numbers close to 1, but larger than 1 and then substitute numbers close to 1, but smaller than 1. For example,

   $$\begin{aligned}f(1.1) &= -0.238230\\ f(1.01) &= -0.248757\\ f(1.001) &= -0.249875\\ f(0.9) &= -0.263340\\ f(0.99) &= -0.251257\\ f(0.999) &= -0.250125.\end{aligned}$$

   A very good guess is that the limit is -0.25.

4. (32 pts)

(a) (8 pts)

$$\begin{aligned}\lim_{x\to 3}(\sqrt{x^2+7}-2) &= \sqrt{3^2+7}-2\\ &= \sqrt{16}-2\\ &= 2.\end{aligned}$$

(b) (8 pts)

$$\begin{aligned}\lim_{x\to 9}\frac{x-9}{\sqrt{x}-3} &= \lim_{x\to 9}\frac{(x-9)(\sqrt{x}+3)}{(\sqrt{x}-3)(\sqrt{x}+3)}\\ &= \lim_{x\to 9}\frac{(x-9)(\sqrt{x}+3)}{x-9}\\ &= \lim_{x\to 9}(\sqrt{x}+3)\\ &= \sqrt{9}+3\\ &= 6.\end{aligned}$$

(c) (8 pts)

$$\begin{aligned}\lim_{x\to 5}\frac{\frac{1}{5}-\frac{1}{x}}{x-5} &= \lim_{x\to 5}\frac{\left(\frac{1}{5}-\frac{1}{x}\right)(5x)}{(x-5)(5x)}\\ &= \lim_{x\to 5}\frac{x-5}{(x-5)(5x)}\\ &= \lim_{x\to 5}\frac{1}{5x}\\ &= \frac{1}{25}.\end{aligned}$$

(d) (8 pts)

$$\lim_{\theta\to 0}\frac{\sin\theta}{\theta} = 1.$$

5. (8 pts) Because the limit is from the right, we need only consider the formula $\frac{x+8}{x}$ as $x \to 4^+$.

$$\begin{aligned}\lim_{x\to 4^+}\frac{x+8}{x} &= \frac{4+8}{4}\\ &= 3.\end{aligned}$$

6. (8 pts) The formula $\frac{x^2-16}{x-4}$ is continuous on $(-\infty, 4)$ and the formula $\frac{x+8}{x}$ is continuous on $(4, \infty)$. The only question is what happens at the endpoint 4?

We have already considered the limit from the right and found that it is 3. But $f(4) = 8$, so $f(x)$ is not continuous on $[4, \infty)$. Now consider the limit from the left.

$$\begin{aligned}\lim_{x \to 4^-} \frac{x^2 - 16}{x - 4} &= \lim_{x \to 4^-} \frac{(x-4)(x+4)}{x-4} \\ &= \lim_{x \to 4^-} (x+4) \\ &= 8,\end{aligned}$$

which does equal $f(4)$. Therefore, $f(x)$ is continuous on $(-\infty, 4]$ and on $(4, \infty)$.

7. (10 pts)

$$\begin{aligned}f'(x) &= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{((x + \Delta x)^2 + 8(x + \Delta x)) - (x^2 + 8x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{(x^2 + 2x\Delta x + (\Delta x)^2 + 8x + 8\Delta x) - (x^2 + 8x)}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{2x\Delta x + (\Delta x)^2 + 8\Delta x}{\Delta x} \\ &= \lim_{\Delta x \to 0} \frac{\Delta x(2x + \Delta x + 8)}{\Delta x} \\ &= \lim_{\Delta x \to 0} (2x + \Delta x + 8) \\ &= 2x + 8.\end{aligned}$$

Now evaluate $f'(x)$ at $x = 1$: $f'(1) = 2(1) + 8 = 10$.

Or you could evaluate the limit

$$f'(1) = \lim_{\Delta x \to 0} \frac{f(1 + \Delta x) - f(1)}{\Delta x}$$

and get the same answer.

8. (10 pts) We are given the point $(2, 4)$. All we need is the slope of the line, which we will get from the derivative.

$$f'(x) = 6x^2 - 10x + 4.$$

Then the slope is $f'(2) = 24 - 20 + 4 = 8$. So the equation of the tangent line is $y - 4 = 8(x - 2)$ or $y = 8x - 12$.